Solving with conjugate method

This method can only be used when either the numerator or denominator contains exactly two terms. Needless to say, its usefulness is limited. Here's an example of a great, and common candidate for the conjugate method.

$$\lim_{h\to 0}\frac{\sqrt{4+h}-2}{h}$$

In this example, the substitution method would result in a 0 in the denominator. We also can't factor and cancel anything out of the fraction. Luckily, we have the conjugate method. Notice that the numerator has exactly two terms, $\sqrt{4+h}$ and -2.

Conjugate method to the rescue! In order to use it, we have to multiply by the conjugate of whichever part of the fraction contains the radical. In this case, that's the numerator. The conjugate of two terms is those same two terms with the opposite sign in between them.

Notice that we multiply both the numerator and denominator by the conjugate, because that's like multiplying by 1, which is useful to us but still doesn't change the value of the original function.

Example

Evaluate the limit.

$$\lim_{h\to 0}\frac{\sqrt{4+h}-2}{h}$$

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Multiply the numerator and denominator by the conjugate.

$$\lim_{h\to 0} \frac{\sqrt{4+h}-2}{h} \cdot \left(\frac{\sqrt{4+h}+2}{\sqrt{4+h}+2} \right)$$

Simplify and cancel the h.

$$\lim_{h\to 0} \frac{(4+h)+2\sqrt{4+h}-2\sqrt{4+h}-4}{h(\sqrt{4+h}+2)}$$

$$\lim_{h\to 0} \frac{(4+h)-4}{h(\sqrt{4+h}+2)}$$

$$\lim_{h\to 0} \frac{h}{h(\sqrt{4+h}+2)}$$

$$\lim_{h\to 0} \frac{1}{\sqrt{4+h}+2}$$

Since we're evaluating at 0, plug that in for h and solve.

$$\frac{1}{\sqrt{4+0}+2}$$

$$\frac{1}{2+2}$$

$$\frac{1}{4}$$

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Remember, if you're trying to evaluate a limit and substitution, factoring, and conjugate method all don't work, you can always go back to the simple method of plugging in a number very close to the value you're approaching and solve for the limit that way.

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